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OVERVIEW OF PERTURBATIVE QCD[†]

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Abstract

I review theoretical techniques and current issues in perturbative QCD, primarily as applied to jet physics at colliders.

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Introduction

In the not-too-distant past, a talk such as this would have discussed “tests of QCD.” Perturbative QCD passed those tests. What, then, is its future in collider physics? As we all know, the ultimate fate of yesterday’s hot physics, at least in particle physics, is to become tomorrow’s background to newer, and presumably hotter, physics. Is this the destiny of perturbative QCD?

To a certain extent, this has already happened. In the analysis leading up to the unveiling of the top quark, the single-lepton channel — where one of the top-antitop pair decays hadronically, while the other’s daughter W boson decays leptonically — played a crucial role. The analysis in this channel requires a careful study of the QCD backgrounds arising from jet production in association with a lone W .

But rather than merely fighting the QCD backgrounds, we may hope to use jet physics as a tool in searching for new physics. Refining our theoretical and experimental techniques with this idea in mind will be one of the important challenges in the coming decade leading up to the commissioning of the LHC. In addition, we may hope to use jet physics to extract information about nonperturbative quantities, such as the parton distribution functions of the nucleon, that remain beyond the present reach of lattice calculations.

Within the context of hadron colliders, experimenters are pursuing studies of a wide variety of jet-associated final states: pure jet production, production of photons or electroweak vector bosons in association with jets, inclusive production of heavy quarks, production of heavy-quark mesons, and production of quarkonia. Different distributions have applications to detailed studies of standard-model observables, such as the mass of the top quark or the mass of the W boson. They also have applications to measurements of parton distributions of the nucleon, especially the gluon distribution, as well as to searches for higher-dimension operators (such as might arise from compositeness or the presence of heavy colored particles in a shorter-distance theory), and to searches for speculative extensions of the standard model.

Jet studies at ep machines can also be useful sources of information about parton distributions, while those at high-statistics e^+e^- machines will offer, upon completion of the next generation of theoretical calculations, a precise measurement of the strong coupling α_s .

Refining jet physics as a tool for doing physics at colliders will require a great deal of theoretical work: in one-loop matrix element calculations; in writing general-purpose, fully-differential numerical programs for a larger number of processes; in setting up a framework for giving honest estimates of errors in predictions due to uncertainties in the extraction of parton distributions; in performing yet-higher order calculations in order to give honest theoretical error estimates. It will also require experimenters to focus on measuring and analyzing those observables which can be predicted most reliably in perturbation theory.

Next-to-Leading Order Calculations

Leading-order calculations perturbative QCD rely only on tree-level matrix elements. These calculations provide a basic description of cross sections and distributions, but are sensitive to potentially large, but uncalculated logarithms. In addition, other aspects of jets, such as their internal structure, cannot be calculated at all in a leading-order program.

The presence of ultraviolet logarithms is reflected in the residual renormalization-scale dependence of a perturbative prediction. Truncating a perturbative expansion at finite order introduces a spurious dependence of a physical observable on the renormalization scale μ_R . Next-to-leading order (NLO) calculations can, and in practice usually do, reduce this dependence significantly compared with leading-order ones. At leading order, the only dependence on μ_R comes from the resummation of logarithms in the running coupling $\alpha_s(\mu_R)$, and the scale choice is arbitrary. At NLO, in contrast, the virtual corrections to the matrix element introduce a separate dependence on μ_R .

As an aside, I would like caution against the common practice of assigning a theoretical “error” by varying the scale up and down by a certain factor (typically two). While the variations do demonstrate the existence of an uncertainty due to theory, the only correct way to assign a sensible error is to compare an NLO calculation (which should already be reasonably stable with respect to variations of the scale), with a yet-higher order calculation. Such estimates will require NNLO calculations.

The other logarithms that arise in perturbative calculations are infrared ones, associated in perturbation theory with the emission of soft or nearly-collinear radiation. In a leading-order calculation, each jet is modelled by a lone outgoing parton. An NLO calculation, however, includes contributions with emission of real radiation. In these contributions, some jets can be made up of two partons. As jet-defining parameters, such as the cone size ΔR , are varied, differing fractions of these contributions will show up as contributions to n - or $(n + 1)$ -jet cross sections. This allows the theoretical prediction to acquire the logarithmic dependence on jet-defining parameters exhibited by experimental data. In addition, the real radiation also gives the leading approximation to a jet’s internal structure.

Calculations

At leading order, each jet is modelled in perturbation theory by a lone outgoing parton. A theoretical prediction of an n -jet distribution in hadron-hadron collisions then requires the probability of finding a parton of given momentum fraction x inside the nucleon, given by the parton distribution function $f_{a \leftarrow p}(x, \mu)$, along with knowledge of the strong coupling $\alpha_s(\mu)$ and the $2 \rightarrow n$ tree-level matrix elements. It also requires a perturbative approximation to the experimental jet algorithm. Assembling these ingredients, the differential distribution in

the experimental observable $X(\{k_j\}_{j \in \text{jets}})$ is

$$\begin{aligned} \left. \frac{d\sigma_n^{\text{LO}}}{dX} \right|_{\text{cuts}} &= \int dx_1 dx_2 \sum_{ab} \int_{\text{cuts}} d\text{LIPS}(x_1 x_2 s; \{k_i\}_{i=1}^n) \\ &\quad \times \alpha_s^n(\mu) f_{a \leftarrow p}(x_1, \mu) f_{b \leftarrow \bar{p}}(x_2, \mu) \text{JetSelect}(\{k_i\}_{i=1}^n) \\ &\quad \times \mathcal{M}(a + b \rightarrow \{f_i\}_{i=1}^n) \delta(X - X(\{k_i\})) , \end{aligned} \quad (1)$$

where $d\text{LIPS}$ is the Lorentz-invariant phase space measure, \mathcal{M} is the tree-level squared perturbative matrix element, and JetSelect is the perturbative approximation to the experimental jet algorithm.

In order to study processes with more exclusive final states, such as processes with specific mesons, we also need a set of *fragmentation functions*, which play the opposite role from distribution functions. The fragmentation function $D_{H \leftarrow a}(z, \mu)$ gives the probability of producing a final-state hadron H with momentum fraction z from the outgoing parton a . It depends on a renormalization scale μ , which in this context is called a factorization scale. Including the fragmentation function, we would obtain a formula along the lines of

$$\begin{aligned} \left. \frac{d^3\sigma_n^{\text{LO}}}{dk_H^3} \right|_{\text{cuts}} &= \int dx_1 dx_2 \sum_{ab} \int_{\text{cuts}} d\text{LIPS}(x_1 x_2 s; \{k_i\}_{i=1}^n) \\ &\quad \times \alpha_s^n(\mu) f_{a \leftarrow p}(x_1, \mu) f_{b \leftarrow \bar{p}}(x_2, \mu) \text{JetSelect}(\{k_i\}_{i=1}^n) \\ &\quad \times \mathcal{M}(a + b \rightarrow \{f_i\}_{i=1}^n) \sum_c \int dz D_{H \leftarrow c}(z, \mu) \delta^3(k_H - z k_1) . \end{aligned} \quad (2)$$

At next-to-leading order, we must combine real-emission contributions with virtual contributions. Each of these contributions is independently singular. This means that we have to combine the contributions analytically, while performing the phase-space integrals numerically. There are two basic approaches to this problem. One, the so-called ‘slicing’ method, is to separate the real-emission phase into two regions. In the soft or collinear region, the integral is calculated analytically, using [universal] soft or collinear approximations. In the remaining region, the integral is finite and can be calculated numerically. The result of integrating the real-emission contribution over the soft and collinear phase space can be combined with the virtual contribution; the sum is again finite, and the integral over [hard] phase space can be performed numerically. For the general version of this approach, see refs. [1].

In the other approach, one subtracts an approximation to the real-emission matrix element everywhere in phase space. The approximation is designed so that the integral factors into an analytically doable (but singular) integration times a phase space integral which can be evaluated numerically. The singularities again cancel the singularities in the virtual corrections to the matrix element. The integration of the original matrix element less the subtrahend is finite, and again can be performed numerically. The reader will find general versions of this approach in refs. [2,3].

Within the general slicing method, one can schematically write the NLO form of a differential cross section as follows,

$$\begin{aligned}
\left. \frac{d\sigma_n^{\text{NLO}}}{dX} \right|_{\text{cuts}} &= \int dx_1 dx_2 \sum_{ab} \alpha_s^n(\mu) \left\{ f_{a \leftarrow p}(x_1, \mu) f_{b \leftarrow \bar{p}}(x_2, \mu) \hat{\sigma}^{\text{LO}}(x_{1,2} \rightarrow n) \right. \\
&\quad + \alpha_s(\mu) f_{a \leftarrow p}(x_1, \mu) f_{b \leftarrow \bar{p}}(x_2, \mu) K(x_1, x_2) \otimes_{\text{perm}} \hat{\sigma}^{\text{LO}}(x_{1,2} \rightarrow n) \\
&\quad + \alpha_s(\mu) [C_{a \leftarrow p}(x_1, \mu) f_{b \leftarrow \bar{p}}(x_2, \mu) + f_{a \leftarrow p}(x_1, \mu) C_{b \leftarrow \bar{p}}(x_2, \mu)] \hat{\sigma}^{\text{LO}}(x_{1,2} \rightarrow n) \\
&\quad + \alpha_s(\mu) f_{a \leftarrow p}(x_1, \mu) f_{b \leftarrow \bar{p}}(x_2, \mu) \hat{\sigma}^{\text{LO}}(x_{1,2} \rightarrow (n+1) \text{ finite}) \\
&\quad \left. + \alpha_s(\mu) f_{a \leftarrow p}(x_1, \mu) f_{b \leftarrow \bar{p}}(x_2, \mu) \hat{\sigma}^{\text{NLO finite}}(x_{1,2} \rightarrow n) \right\}. \tag{3}
\end{aligned}$$

In this equation, the second term inside the braces summarizes the contribution of the integration over soft and final-state collinear regions, once combined with the corresponding virtual singularities. The third term summarizes the contribution of initial-state collinear regions, again along with corresponding virtual singularities. It makes use of *crossing functions* C_a , which are essentially convolutions of the parton distribution functions with the Altarelli-Parisi splitting functions. These functions, along with the K function in the second term, are independent of the short-distance process, and hence do not have to be calculated anew for each new process one wants to calculate numerically. The fourth term gives the contributions of the finite parts of the real emission contributions, that those outside the soft and collinear regions. The last term gives the contributions of the finite parts of the virtual corrections to the matrix element; it is the complexity of calculating these corrections that is at present the limiting factor in writing NLO programs for new processes.

Single-Jet Inclusive Distribution

One of the simplest distributions to consider is the single-jet inclusive distribution. As its name suggests, one is studying

$$p\bar{p} \rightarrow \text{jet} + X,$$

binning all jets in transverse energy E_T . Of course, most events that show up in this distribution are actually two-jet events (the second jet is needed to balance the E_T). Now, production of a pair of jets at high E_T requires a large partonic center-of-mass energy, and so the high- E_T tail of this distribution probes the large- \hat{s} region, where one is most likely to see signals of new physics.

The CDF collaboration has claimed to find evidence of a discrepancy⁴⁾ between their results and an NLO calculation⁵⁾. The nature of the discrepancy depends on which parton distribution set is used in the theoretical calculation. CDF chose to use an older set, and then found that data points at transverse energies of 250 GeV and up are systematically higher than the data.

Were α_s known to much higher accuracy than it is, were the gluon distribution in the proton known to much higher accuracy than it is; and were the discrepancy a remarkable rise

at large- E_T , well outside of statistical and systematic errors, one might then lean towards seeing in it a signal of physics beyond the standard model, perhaps indeed of compositeness. Unfortunately, neither α_s nor the gluon distribution are known well enough to draw such conclusions, and the other stated hypotheses deserve closer scrutiny as well.

The DØ collaboration’s results, as presented at this conference by G. Blazey⁶⁾, fail to confirm the CDF claim. They don’t necessarily contradict it, either; a more careful comparison of the two data sets and a more thorough examination of the systematic errors of the DØ data set would be required to draw such a conclusion.

This differential cross section spans an enormous dynamic range, from 10^4 nb/GeV at $E_T \sim 60$ GeV to 10^{-2} nb/GeV at $E_T \sim 400$ GeV. We must bear in mind that when we view the experimental results in the form $(\text{data} - \text{theory})/\text{theory}$, certain systematic errors can induce rather large effects. In particular, the experimental data must be corrected for the detector’s resolution: a real-world detector may report an energy deposit different from the actual energy of a jet. Correcting for this resolution requires shifting jets along the distribution from one E_T to another; the rapidly-falling distribution magnifies the results of uncertainties in estimating the tails of the resolution function.

Aside from possible experimental systematic errors, the most plausible explanation for the discrepancy is our lack of sufficiently detailed knowledge of the gluon distribution function in the nucleon. As S. Kuhlmann showed in his talk⁷⁾, the use of a different gluon distribution, along with a slightly different $\alpha_s(M_Z)$, will bring the QCD prediction into agreement with the CDF results. (The modified distribution and $\alpha_s(M_Z)$ still agree with deeply inelastic scattering data.)

Other theoretical “explanations” of the excess seem to me much less plausible. Even at the highest energies in the CDF distribution, x_T is at most of order 0.5. These points are thus far from the kinematic endpoint, and resummation of end-point logarithms (of the generic form $\alpha_s \ln(1 - x_T)$) seem unlikely to contribute a 50% effect. Higher-order corrections also seem an unlikely candidate; while the NLO-to-LO ratio depends sensitively on the way the renormalization scale is chosen, with a natural scale choice of $\mu \sim \mathcal{O}(E_T)$, this ratio is flat over a wide E_T range, and thus cannot explain the change of *shape* the CDF data seemingly require.

It will, of course, be interesting to see other distributions — such as the dijet angular distribution — which would generically differ substantially from QCD predictions at high E_T were new physics to show up.

Quarkonia

Charmonium, and to a lesser extent bottomonium, production at hadron colliders are potentially useful probes. Charmonium production also plays an important role in collider studies of B physics, which are in turn promising for studies of CP violation.

For many years, theorists assumed that charmonium production was dominated by perturbative $c\bar{c}$ production⁸⁾. CDF data disagreed wildly with these expectations. Braaten and Yuan showed, however, that at large transverse momentum, fragmentation is actually the dominant production process⁹⁾. Recent CDF data, presented by V. Papadimitriou¹⁰⁾, now distinguish between direct J/Ψ production, and production from B decay. These data still show a much larger rate than would be predicted from the fragmentation contribution assuming the latter is dominated by production of color-singlet charmonium states. Including color-octet production¹¹⁾ seems to bring the theory into much better agreement with the data at the Tevatron, though it is not yet clear that we have obtained a picture consistent with HERA data¹²⁾.

Prompt Photons

The study of prompt photon production,

$$p\bar{p} \rightarrow \gamma + X,$$

is in principle a good way to extract information about the gluon distribution in the proton¹³⁾. In practice, it has been plagued by problems concerning an appropriate experimental and theoretical definition of “photons”. Because of potential problems with contamination from $\pi^0 \rightarrow \gamma\gamma$ with overlapping photons inside the detector, experimenters do not try to observe photons inside jets. Instead, they demand that photons be isolated away from jets.

However, from a theoretical point of view, a total isolation cut (no hadronic energy inside a cone surrounding the candidate photon direction) is a bad idea, because such a cross section is divergent in perturbation theory. (Chopping out a cone in phase space prevents the real-emission contributions from cancelling all of the singularities in the virtual corrections.) As a result, such a cross section is very sensitive to long-distance, that is non-perturbative, physics.

A theoretically more satisfactory approach is to restrict the hadronic energy *fraction* inside the cone, though recent papers have raised questions about the detailed cancellations here as well¹⁴⁾. Even with the more theoretically satisfactory definition, however, the measurements¹⁵⁾ seem to lie above the theoretical predictions¹⁶⁾ at E_T below 30 GeV. In the case of the DØ data, one may be tempted to ascribe the disagreement to the larger experimental systematic errors at low p_T , but for the CDF data this doesn’t work. Adding parton showering¹⁷⁾ to, or putting in an intrinsic k_T into the theoretical calculation⁷⁾ (both in an ad hoc way) brings the predictions into better agreement with the data. While this may provide clues to a resolution of this discrepancy, it cannot be considered satisfactory in itself.

Jet Algorithms

Measuring jet cross sections requires a precise definition of a jet. A jet algorithm, as used

by experimenters, must specify how to cluster the sprays of hadrons observed in the detector into jets. It must also have a matching version to be used by theorists, which specifies how to cluster partons in a perturbative calculation into jets.

In principle, any infrared-safe algorithm could be used to compare experimental data with perturbative calculations. In order to make the best use of data, however, it is best to choose an algorithm with good theoretical properties, in particular with small higher-order corrections.

In e^+e^- annihilation, such considerations have played an important role in the shift from the traditional JADE or invariant-mass algorithm to the so-called k_\perp or Durham algorithm¹⁸⁾. The latter allows resummation, and is expected to have smaller power hadronization corrections and better mass resolution than the JADE algorithm¹⁹⁾.

In hadron-hadron collisions, in contrast, both collaborations use variants of the so-called ‘Snowmass’ cone algorithm²⁰⁾. While this algorithm presumably has better properties than the JADE algorithm, there are aspects of it that are poorly modelled in low orders of perturbation theory. In particular, in the experimental algorithm one is sometimes faced with the choice of ‘splitting’ a jet which contains two distinct centers. This cannot be modelled in an NLO calculation (the simplest perturbative approximation requires three partons forming the proto-jet, hence an NNLO calculation), and is thus a source of uncertainty in the theoretical calculation. The hadronic version of the k_T algorithm²¹⁾ avoids this problem, because the η - ϕ plane is not split up into rigid circles as in the cone algorithm, but rather into odd-shaped regions that adapt to the shapes of the jets in a given event. This algorithm presumably shares many of the features of its e^+e^- forbear, such as better mass resolution²²⁾.

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